No. of Printed Pages : 5

Sem-II-Math-GE-B_1(R&B)

2022

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

GROUP - A

1. Fill in the blanks. (all) [1 × 12

(a) The number of loops in the curve $r^2 = a^2 \cos 2\theta$ is _____.

(b) The curve of nth degree and its asymptotes intersect each other at _____ points.

(c) $\lim\limits_{x \to \infty} \frac{x}{[x]}$ = _____.

(d) The curve $x^{2/3} + y^{2/3} = a^{2/3}$ is symmetrical about _____ axis.

(e) $\lim\limits_{(x,y) \to (0,0)} \frac{xy^4}{x^2 + y^2}$ = _____.

P.T.O.

(f) If $f(x, y) = \frac{xy}{x+y}$ then $f_x(2, 1) =$ _____

(g) If $f(x) = x + [x]$, then $f'(2^-) =$ _____.

(h) The function $f(x) = |2x - 3|$ is discontinuous at _____.

(i) If $u = x^3 + y^4$ where $x = t^2$ and $y = t^3$, then $\frac{du}{dt} =$ _____

(j) The integrating factor of $y(axy + e^x)dx - e^x dy$ is _____.

(k) The P.I. of $(D^2 + 1)y = \cos x$ is _____.

(l) The Wronskian $W(y_1, y_2)$ of $(D^2 + 1)y = \text{cosec } x$ is _____

GROUP - B

2. Answer any eight questions. [2 ×

(a) Determine where the loop of the curve $3ay^2 = x(x - a)^2$ lie

(b) Find the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(c) Find the radius of curvature of the curve

$y = 4 \sin 2x - \sin 4x$ at $x = \pi/4$.

(d) Write L' Hospital's Rule.

(e) Write Rolle's theorem.

(f) If $u = x^2 + xy + y^2$, $x = r + t$, $y = r - t$, then find $\frac{\partial u}{\partial t}$.

(g) If $u = x^2y$, where $x^2 + xy + y^2 = 1$, then find $\frac{du}{dx}$.

(h) Find P.I. of $(D - 2)^2y = e^{2x}$.

(i) Solve : $y = px + \frac{a}{p}$.

(j) Solve : $y^2 + p^2 = a^2$

GROUP - C

3. Answer any eight questions. [3 × 8

(a) Find the centre and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 - 4x + 4y - 8z + 10 = 0$$

(b) Trace the curve $r = a$

(c) Find $\lim_{x \to 0} \sin \frac{1}{x}$

(d) Find $\lim_{x \to 0} x^{\sin x}$

P.T.O.

(e) Find the Maclaurin's series of $f(x) = \log(1 + x)$.

(f) Show that the equation $10x^4 - 6x + 1 = 0$ has a root between 0 and 1.

(g) Show that the function $x^4 + x^2y + y^2$ has a minimum at $(0, 0)$.

(h) Find $\frac{d^2y}{dx^2}$, if $x^3 + y^3 - 3axy = 0$.

(i) Find the P.I. of $(D^2 + 1)y = \text{cosec } x$ using the method of variation of parameter.

(j) Find the P.I. of $(D^2 + 4)y = x \sin x$

GROUP - D

4. Answer any four questions. [7 × 4

(a) Prove that the length of the loop of the curve $3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$.

(b) State and prove Taylor's theorem with Cauchy's form of remainder.

(c) If $u = \tan^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$.

(d) If $f(x, y) = \begin{cases} \dfrac{xy\left(x^2 - y^2\right)}{x^2 + y^2} & , \ (x,y) \neq (0,0) \\ 0 & , \ (x,y) = (0,0), \end{cases}$

then show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

(e) Show that IVP $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 2e^x$, $y(0) = 1$, $y'(0) = 1$.

(f) Solve $p^2x^2 - 2xyp + 2y^2 - x^2 = 0$.

(g) State and prove Taylor's theorem with Lagrange's form of remainder.