No. of Printed Pages : 5

Sem-I(Arts&Sc)-Math-CC-II (Reg&Back)

2020-21

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Symbols used have their usual meaning.

GROUP – A

1. Answer all questions or fill in the blanks as required. [1 × 12

(a) $5 \in \{x + 2y : x \in \{0, 1, 2\}, y \in \{-2, 0, 2\}\}$. Write true or false.

(b) Find A^C (with respect to R), where $A = (1, \infty) \cup (-\infty, -2]$.

(c) If gcd(a, b) = 1, then lcm(a, b) = ab. Write true or false.

(d) If $a_1 = 5$ and $a_{k+1} = 3a_k$ for $k \geq 1$, then $a_3 =$ ______ .

(e) If $n \in N$, then $P(n, 1) =$ ______ .

(f) The expansion of $(3x - \frac{5}{x})^8$ has a constant term. Write true or false.

(g) If A is involuntory, then $A^2 =$ ______ .

(h) If A has two identical rows, then det A = ______ .

P.T.O.

(i) If λ is an eigenvalue of A, then the eigenvalue of A^2 is ______.

(j) The number of edges of K_5 is ______.

(k) Two edges are said to be adjacent if they have a ______ in common.

(l) Write the adjacency matrix of K_2.

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Show that $p \wedge (\neg p)$ is a contradiction.

(b) Prove that $A \cap A = A$.

(c) Show that $f : R \to (1, \infty)$ and $g : (1, \infty) \to R$ defined by

$$f(x) = 3^{2x} + 1,\ g(x) = \frac{1}{2}\log_3 (x - 1)$$ are inverses.

(d) Given three consecutive integers a, a + 1, a + 2. Prove that one of them is divisible by 3.

(e) A man. woman, boy, girl, dog and cat are walking down a long and winding road one after the other. In how many ways can this happen if the dog comes first ?

(f) Find A and B if $A + B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

(g) For all sets A and B, if $A \subseteq B$ then prove that $P(A) \subseteq P(B)$.

(h) By elementary row transformations, find A^{-1},

where $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.

(i) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$.

(j) Define a pseudograph. Give one example.

(k) Draw the graph $W_5 - C_5$.

(l) Define a bridge. Give one example.

GROUP – C

3. Answer any eight of the following. [3 × 8

(a) Define $f : Z \to Z$ by $f(x) = x^2 - 5x + 5$. Verify f is one-one and / or onto.

(b) Show that (–1, 2) and (–5, 4) have the same cardinality.

(c) If $a \equiv x \pmod{n}$ and $b \equiv y \pmod{n}$, then prove that

$$a + b \equiv x + y \pmod{n} \text{ and } ab \equiv xy \pmod{n}.$$

(d) How many solutions are there of $x + y + z = 17$ subject to constraints $x \geq 1, y \geq 2, z \geq 3$?

(e) Show that in any set of eleven integers, there are two whose difference is divisible by 10.

(f) Find x, y, z and t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$.

(g) Show that the sequence $\{2, 3, 4, 5, \ldots\ldots\ldots, 2 + n, \ldots\ldots\ldots\}$ for $n \geq 0$ satisfies the recurrence relation $a_k = 2a_{k-1} - a_{k-2}$, $k \geq 2$.

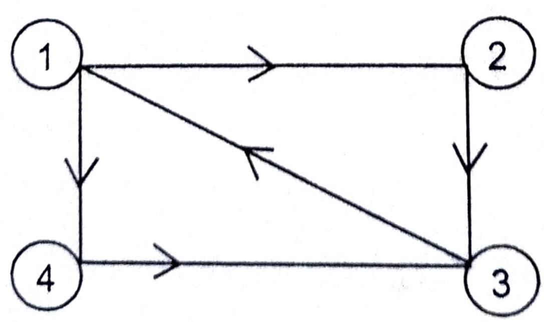
(h) Find the minors and cofactors of each element of 2nd row of the determinant $\begin{vmatrix} 1 & 2 & 0 \\ 1 & 5 & 1 \\ 3 & 7 & 1 \end{vmatrix}$.

(i) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$, verify that $A(\text{adj } A) = |A| I$.

(j) Is there a simple graph corresponding to the degree sequence (2, 2, 4, 6) ? Justify your answer.

(k) Show that C_6 is a bipartite graph.

(l) Is the given graph strongly connected ?



GROUP – D

*Answer **any four** questions.*

4. Show that [7

$$[(p \vee q) \vee ((q \vee (\neg r)) \wedge (p \vee r))] \Leftrightarrow \neg[(\neg p) \wedge (\neg q)].$$

5. Let $a, b \in Z$, $b \neq 0$. Then show that there exist unique integers q and r, with $0 \leq r < |b|$, such that $a = qb + r$. [7

6. Solve the recurrence relation [7

$$a_n = 5a_{n-1} - 2a_{n-2} + 3n^2,\ n \geq 2, \text{ given } a_0 = 0,\ a_1 = 3.$$

7. Find, how many integers between 1 and 60 that are not divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5, not by 2, not by 3. [7

8. Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. [7

9. Solve the equations $5x + 3y + 7z = 4$, $3x + 2y + 26z = 9$, $7x + 2y + 10z = 5$ by matrix method. [7

10. Find the eigenvalues and eigenvectors of the matrix [7

$$A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}.$$