No. of Printed Pages : 4

Sem-I(Arts&Sc)-Math-CC-I
(Reg&Back)

2020-21

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Symbols used have their usual meaning.

GROUP – A

1. Answer all questions. [1 × 8

(a) Define tangent hyperbolic function and write down its range.

(b) Find the derivative of log(cos hx).

(c) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$.

(d) Integrate using reduction formula $\int_0^{\pi/2} \sin^8 x \, dx$.

(e) Find the arc length of the spiral $r = e^{\theta}$, between $\theta = 0$ and $\theta = \pi$.

(f) Write the parametric equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(g) Evaluate $\lim_{t \to 0} \left[\frac{t e^t}{1 - e^t} \mathbf{i} + \frac{e^{t-1}}{\cos t} \mathbf{j} \right]$.

(h) Find the acceleration of the moving object represented by vector $R(t) = t \sin t\mathbf{i} + t e^{-t}\mathbf{j} + (t - 1)\mathbf{k}$.

GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) Evaluate $\int \sin^2 hx \,.\, \cos^2 hx \, dx$.

(b) Find Y_n if $Y = \frac{1}{4x + 3}$.

(c) Using reduction formula, evaluate $\int_0^{\infty} \frac{dx}{(1 + x^2)^4}$.

(d) Evaluate $\lim_{x \to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$.

(e) Evaluate $\int_0^1 \left(t^3 \mathbf{i} + \sqrt{t} \mathbf{j} - \cos \frac{\pi}{2} t \mathbf{k}\right) dt$.

(f) Find the asymptote parallel to coordinate axes $xy^2 = (x + y)^2$.

(g) Find the total length of cardioid

$$r = 1 + \cos\theta, \; 0 \le \theta \le \pi.$$

(h) Find the tangent vector to the vector function defined by

$$F(t) = t^2\mathbf{i} + (\cos t)\mathbf{j} + (t^2 \cos t)\mathbf{k} \text{ at } t = 0, \tfrac{\pi}{2}.$$

(i) Find the point of inflexion to the curve $y = (\log x)^3$.

(j) Find the interval for which the curve $f(x) = x^3$ is concave upward and downward.

GROUP – C

3. Answer any eight of the following. [2 × 8

(a) Prove that $\cos h^{-1}x = \ln\left[x + \sqrt{x^2 - 1}\right]$.

(b) If $y = \dfrac{1}{4x^2 - 9}$, find y_n.

(c) Find the ranges of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward or downward. Determine the point of inflexion.

(d) Show that the curve $x^4 + y^4 = a^2(x^2 - y^2)$ has no asymptote.

(e) Find a vector that is orthogonal to both of vectors $u = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $v = -7\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(f) Find the volume of the solid that results when the region above x-axis and below the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $(a > 0, b > 0)$ is revolved about x-axis.

(g) Find the exact length of arc

$x = \cos t + t \sin t,$

$y = \sin t + t \cos t, \quad (0 \le t \le \pi).$

(h) Find the area of the surface generated by revolving

$$x = \cos^2 t,\ y = \sin^2 t,\ \left(0 \le t \le \tfrac{\pi}{2}\right) \text{ about x-axis.}$$

(i) Find the area of the triangle whose vertices are A(2, 2, 0), B(−1, 0, 2), C(0, 4, 3) by using vector method.

(j) Using scalar triple product, find the volume of parallelopiped whose adjacent sides are A(2, −6, 2), B(0, 4, −2), C(1, 2, −4) respectively.

GROUP – D

*Answer **any four** questions.*

4. If $y = a\cos(\log x) + b\sin(\log x)$, show that [6

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

5. Using reduction formula, prove that [6

$$\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}dx = \frac{3\pi}{128}.$$

6. Find the area of the surface generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about x-axis. [6

7. Evaluate $\lim_{x \to 0}\left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{\frac{1}{x}}$. [6

8. Find the asymptote of the curve [6

$$x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0.$$

9. If $r(t) = 5t^2\mathbf{i} + t\mathbf{j} - t^2\mathbf{k}$, evaluate $\int_1^2 [r(t) \times r''(t)]dt$. [6